Measuring Polar Axis Alignment Error

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Abstract: One of the most popular methods for polar alignment is the drift alignment method. In this paper the drift method is examined in detail. It is shown that the declination drift can be used to calculate the polar axis alignment error. Alternative methods for measuring declination drift and alignment error are also given.

The Drift Alignment Method

The drift alignment method is based on the fact that carefully selected stars tend to drift in declination over time if the mount is not properly aligned. Here is how the procedure works. To measure and adjust the azimuth axis, monitor a star near the intersection of the celestial equator and the meridian (an hour angle of 0 hours). If the star drifts north the mount is pointing too far west. A southern drift indicates the mount is pointing too far east. Likewise, to measure and adjust the altitude axis, monitor a star in the east (an hour angle of –6 hours) near the celestial equator. If the star drifts north the mount is pointing too high. A southern drift indicates the mount is pointing too low. In each case the rate of drift is indicative of the magnitude of the error and the adjustment required for correction. Note that the drift direction should be reversed in the Southern Hemisphere or if an altitude reference star is selected in the west (an hour angle of 6 hours). The procedure is repeated on each axis until no discernable drift is observed after a significant amount of time. [1]

How the Drift Method Works

Fundamentally, when a mount is not properly aligned, it is rotating on an axis that is not parallel to the celestial axis created by the earth's rotation. We will examine this by first looking at a mount that is misaligned only in altitude, and then only in azimuth, and see what happens with the mount's movement as compared with the celestial sphere's movement (actually the earth's movement). By examining each of the alignment axes individually it is easier to discover the movements of the mount and how the drift method reveals the alignment error to us.

Altitude Error

To understand the geometry of misalignment it is best to think of what happens within the celestial sphere as a perfectly aligned mount is taken out of alignment. In this case we will look at what happens to the sphere as we move the mounts altitude axis out of alignment by raising or lowering the altitude adjustment. Figure 1 depicts the movements assuming a Northern Hemisphere observer. Notice what happens to the celestial equator as the mount's altitude axis is moved. Moving the axis higher causes the equator to tip lower. Conversely, moving the axis lower causes the equator to tip higher. The mount's movement is nearly parallel to the celestial equator near the meridian, but near the eastern and western horizon the mount's movement differs from the movement on the equator by the angle of misalignment.

Some more illustrations may help clarify this. In Figure 2 we see what happens to a star on the equator near the eastern horizon. At time t_1 we begin our observation. At time t_2 the earth has rotated such that the star now appears westward along the equator. If the altitude axis is too low, the mount will track at a higher angle and will be pointing at X_B. As we observe a star in the eyepiece during this time it will be moving along the equator, but will appear to us to be drifting south due to the mount's upward movement relative to the equator. Thus the rule: If an eastward star drifts south, the mount is pointing too low. If the altitude axis is too high, the mount will track at a lower angle and will be pointing at X_A. This star will appear to drift north due to the mount's downward movement relative to the equator. So if an eastern star drifts north, the mount is pointing too high. Figure 3 shows the situation when looking at a star on the equator near the western horizon. Here the situation is reversed. If the star appears to drift south it indicates that the mount is pointing too high. If the star appears to drift north, it indicates that the mount is pointing too low.

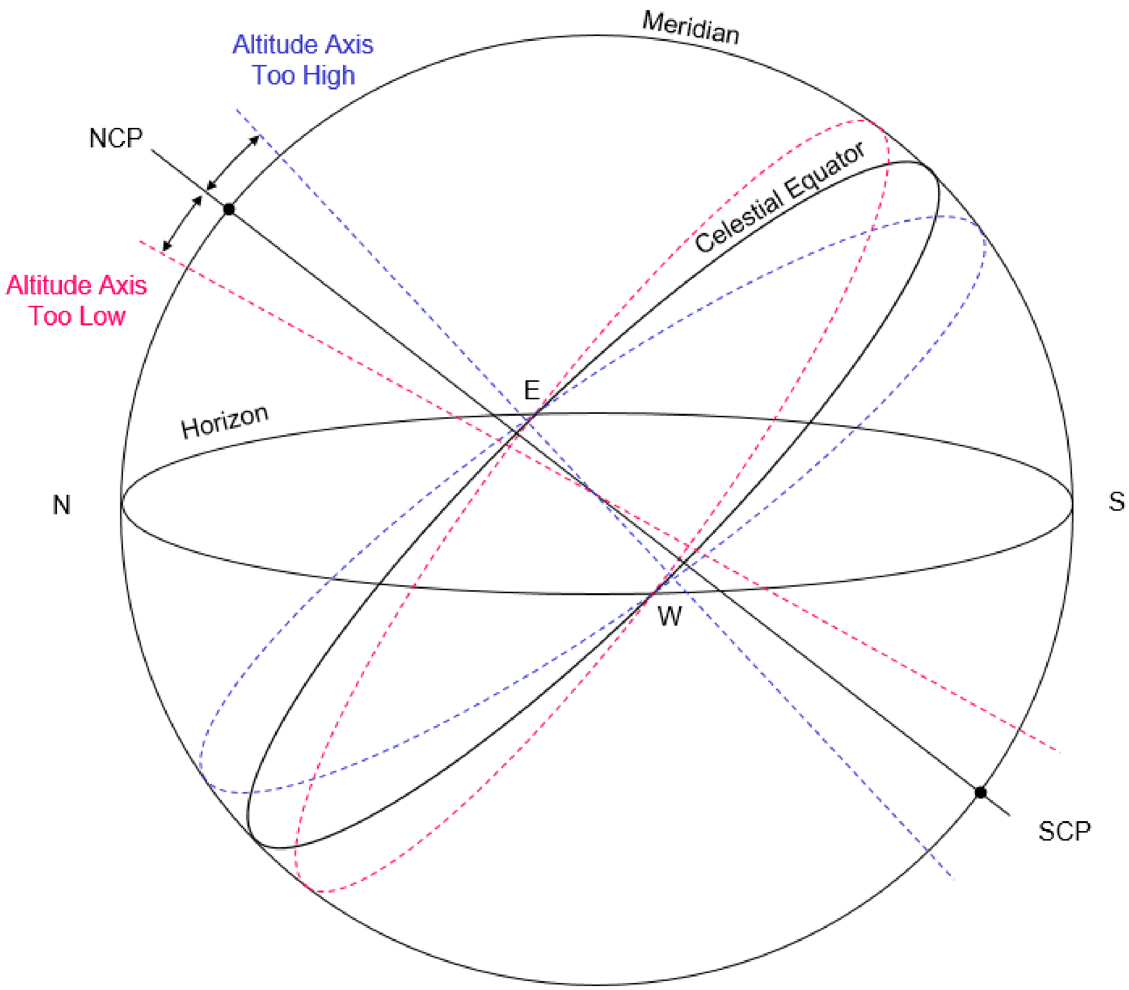

Figure 1. When the mount's altitude axis is moved higher the celestial equator tips downward. Conversely, moving the altitude axis lower tips the equator upward.

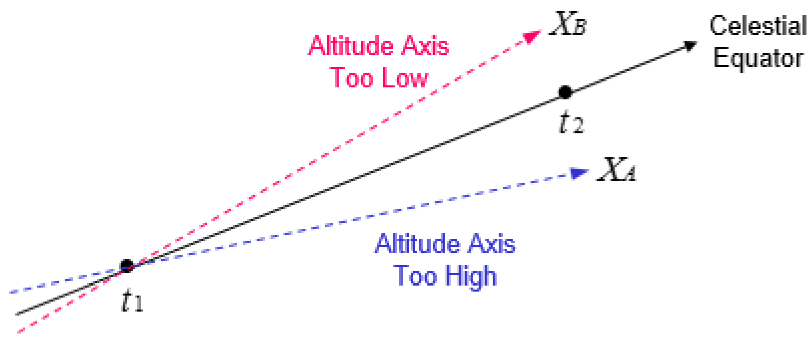

Figure 2. The movement of the mount when observing a star on the equator near the eastern horizon. When the altitude axis is too high the star appears to drift north. If the axis is too low the star appears to drift south.

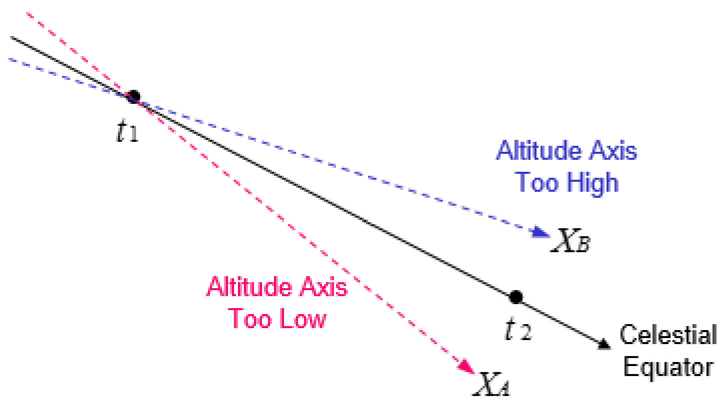

Figure 3. The movement of the mount when observing a star on the equator near the western horizon. When the altitude axis is too high the star appears to drift south. If the axis is too low the star appears to drift north.

Azimuth Error

Figure 4 depicts the movements of a Northern Hemisphere observer when the azimuth axis is shifted toward the east and the west. Notice what happens to the celestial equator as the mount's azimuth axis is moved. When the axis moves eastward the equator tips higher. Conversely, moving the axis westward causes the equator to tip lower. The mount's movement is nearly parallel to the celestial equator near the eastern and western horizon, but near the meridian the mount's movement differs from the movement on the equator by the angle of misalignment.

Again, an illustration may help clarify this. In Figure 5 we see what happens to a star on the equator near the meridian. At time t_1 we begin our observation. At time t_2 the earth has rotated such that the star now appears westward along the equator. If the azimuth axis is too far east, the mount will track at a higher angle and will be pointing at X_B. The effect is that the star will appear to be drifting south due to the mount's upward movement relative to the equator. So if a star on the equator near the meridian drifts south, the mount is pointing too far east. If the azimuth axis is too far west, the mount will track at a lower angle and will be pointing at X_A. This star will appear to drift north due to the mount's downward movement relative to the equator. So if the star drifts north, the mount is pointing too far west.

Optimal Measurement Locations

Notice that the optimal point for observing azimuth error is at the intersection of the equator and the meridian. But due to the orthogonal geometry this is also the location where altitude error is virtually undetectable. Conversely, we observe altitude errors best on the equator at the eastern or western horizon, and in this location azimuth error is minimized. This is how we are able to use the drift method to measure and correct azimuth and altitude errors independently. Of course, this all assumes that the mount base is leveled, as an unleveled base would cause azimuth corrections to affect altitude and vice versa.

It may not always be possible to observe at these optimal locations due to obstructed horizons or the lack of a sufficiently bright star to observe. Measuring altitude error near the horizon is usually discouraged because of errors that can creep in from atmospheric refraction. If we must deviate from these optimal locations, what is the effect on the accuracy of the measurement?

Let us first look at deviations from the celestial equator. At the celestial equator we are at the maximum angular distance from either pole. Therefore, at this location the amount of drift is maximized and tends toward zero as we approach the pole. The following equation shows us how to calculate polar alignment error based on declination drift. It shows that the amount of drift is inversely proportional to the declination by a factor of the cosine of declination: [2]

$$\theta = \frac{12}{\pi} \cdot \frac{\delta_{err}}{t\cos\delta} \cong 3.8197\,\frac{\delta_{err}}{t\cos\delta} \qquad (1)$$

Where:

θ	is the alignment error in arc minutes
δ_{err}	is the declination drift in arc seconds
t	is the time of drift in minutes
δ	is the declination of the drift star used

Note: This equation is formally derived in Appendix A.

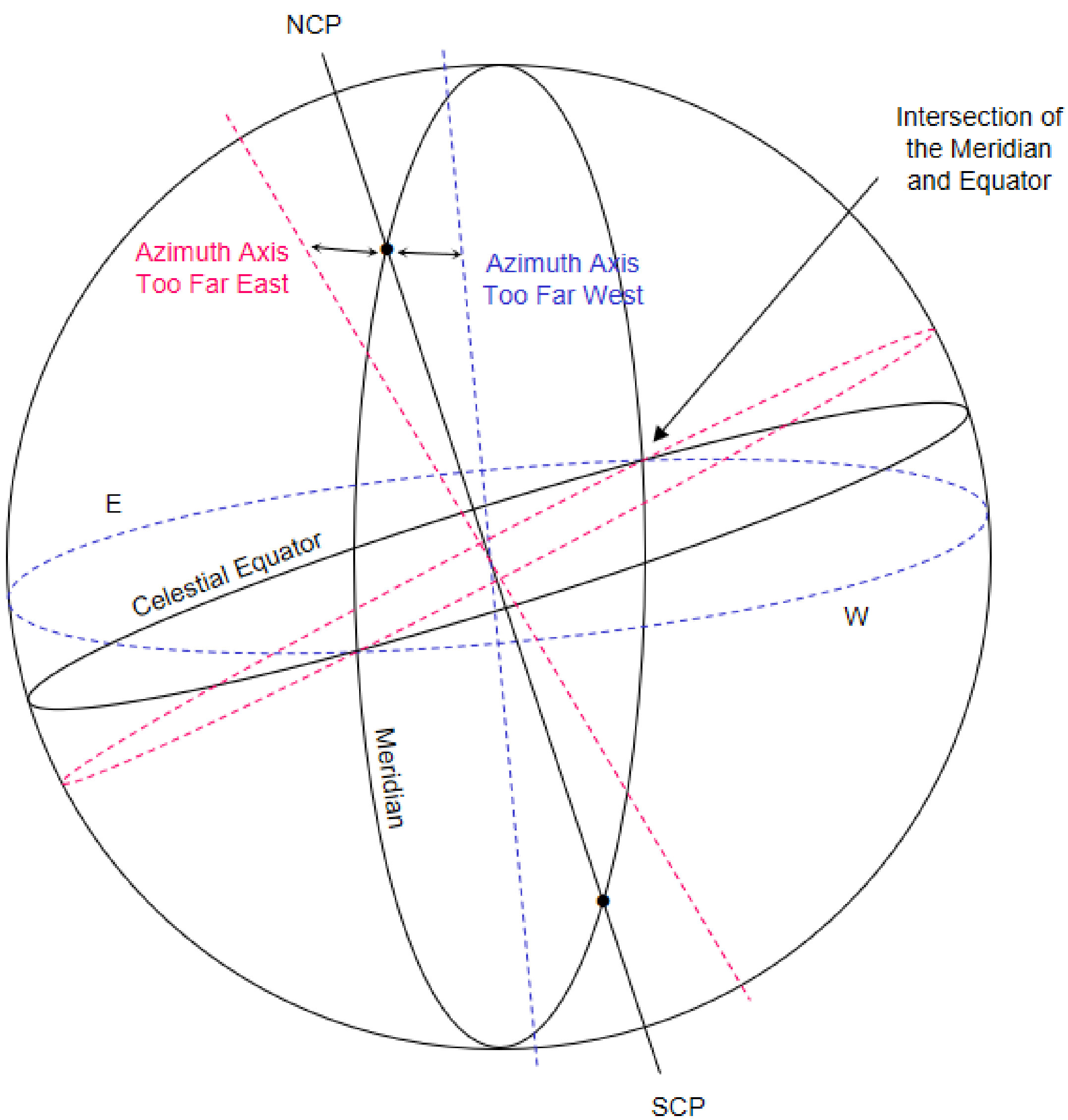

Figure 4. When the mount's azimuth axis is moved to the east, the celestial equator tips upward. Conversely, moving the azimuth axis to the west tips the equator downward.

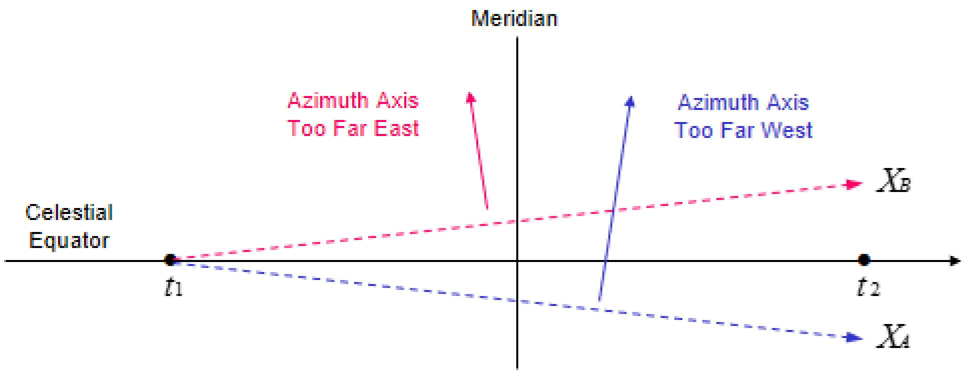

Figure 5. The movement of the mount when observing a star on the equator near the meridian. When the azimuth axis is too far west the star appears to drift north. If the axis is too far east the star appears to drift south.

Deviation from the equator will not result in loss of accuracy as long as the angular distance is not great. However, what about deviating from the optimal hour angle? Here the optimal hour angle is 0 hours (on the meridian) for azimuth measurements and +/- 6 hours (points east and west) for altitude measurements. As we noted earlier the orthogonal geometry permits us to measure and correct each axis independently. But we also observe that as we deviate from these optimal hour angles we begin to approach the opposite axis' optimal hour angle. Therefore, the declination drift measured at non-optimal hour angles will not be due to a single axis error but some combination of both axes. [2]

It is rarely the case that a reference star near the meridian cannot be found. If your horizon is obstructed all the way to the zenith, polar alignment is clearly not your greatest problem! Therefore, we will assume that the azimuth error can be measured and corrected with an optimal hour angle. If we further assume that the azimuth error is correct before measuring and correcting altitude error, we can make some simplifying assumptions about the declination drift when measuring the altitude error. In other words, if we know by previous correction and measurement that the azimuth error is negligible, then we can assume that the declination drift measured in the vicinity of the optimal altitude locations is due entirely to the altitude error. The following equation can be used to correct for altitude error at a non-optimal hour angle when azimuth error is known to be negligible:

$$\theta_{alt} = \frac{\theta}{|\sin H|} \qquad (2)$$

Where:

θ_{alt} is the altitude alignment error

θ is the alignment error calculated by Equation (1)

H is the hour angle where θ is measured

We can see by Equation (2) that when the hour angle is +/- 6 hours the sine is equal to one and thus $\theta_{alt} = \theta$. As the hour angle deviates from the optimal hour angle the function serves to increase the calculated value to compensate. Notice that the function will fail if the deviation is great. At locations near the meridian (hour angle of zero hours) the function tends toward infinity.

Measuring Error Visually

This is the most common way of measuring the polar alignment error when the drift method is used. Most astronomers do not even attempt to measure the error quantitatively, but note the drift direction and drift rate and estimate the adjustment to the mount based on these qualitative notions in a sort of trial-and-error fashion. However, it is beneficial to be able to quantify the error for two very good reasons. First, if the error is measured quantitatively with some degree of accuracy, the adjustment to the mount is more predictable and precise. [3] Second, the alignment error of the mount is needed to determine if the alignment is within tolerances for a given observing or imaging session. [2]

There are reticle eyepieces available which contain etched scales that can be useful for measuring declination drift when calibrated correctly and a sufficiently long focal length is employed. With this approach the declination drift can be estimated with reasonable accuracy. The alignment error for the axis can then be calculated by means of Equations (1) and (2).

Measuring Error with an Autoguider

Autoguiding software programs work by taking images of a reference star and measuring the distance of that star from a reference pixel in the image. Corrections are sent to the mount to reposition the star at the reference point and the process is repeated throughout the duration of the exposure. Most autoguiding software can operate in a mode where the star offset is reported without any correction made to the mount. This is perfect for our purposes since the offset in the declination axis over time is the measurement in which we are interested. Note that a requirement of Equation (1) is that the drift needs to be expressed in arc seconds. If the autoguiding software reports the error in pixels the needed conversion is as follows:

$$\delta_{err} \cong \sigma_{err} \frac{206.265\mu}{F} \qquad (3)$$

Where:

δ_{err} is the calculated drift in arc seconds

σ_{err} is the measured drift in pixels

μ is the pixel size in microns

F is the focal length in millimeters

Dealing with Atmospheric Seeing

Atmospheric seeing can conspire to prevent an accurate reading of declination drift. As the star image moves around due to the atmosphere, the star declination offset value can vary by a few arc seconds from one measurement to the next. One way to minimize this is to take an exposure for several seconds to average out the seeing effect. A reasonable estimate can also be attained by averaging the last few measurements to smooth out the effect. A more statistically correct method is to collect all the measurements and perform a linear regression of the data set. Using the slope of the linear fit, a very accurate estimate can be calculated for the time period desired. Many autoguiding software programs provide the capability to log the offset values to a file. This file can be imported into a spreadsheet program to perform this analysis. However, whatever method is employed, it is important to point out that when the alignment error is small, the variance due to seeing effects is more statistically significant and can negatively impact the accuracy of the measurement. The best way to mitigate against this is to take measurements for longer time periods when the drift rate is low.

Measuring Error by Moving in Right Ascension

One of the drawbacks of the classic drift method is that high precision is very time demanding. The problem is easily solved by close examination of Figures 2, 3 and 5. In these figures we are noting the deviation of the mount's movement as compared to the earth's movement. The star at t_1 moves to a location at t_2 after some time period and the declination drift and its direction are measured. However, we need not wait on the earth to rotate to measure this discrepancy! If we can locate two stars of reasonably close declination near the optimal measurement location, then simply aligning on the first star and moving to the second star's coordinates should also reveal to us the declination deviation. Since a misaligned mount will be moving on an angle above or below the movement of the earth, the second star will not be centered after the movement. The movement in declination that is required to recenter the second star is the deviation we are seeking! This method requires that we have the ability to measure declination accurately to the arc second, requiring either accurate digital setting circles or a mount with pointing capability. Here is the procedure:

1. Center the first reference star. Make sure that the mount is synchronized to the coordinates of this star. It is also important that the mount has disabled its pointing model for maximum accuracy. If the mount is already compensating for pointing errors the procedure will fail.
2. Move the mount to point at the coordinates of the second reference star. The second reference star need not be exactly on the same line of declination as the first, but smaller movements in declination will prevent errors from creeping into the measurement.
3. Center the second reference star and note where the mount thinks it is pointing now. The discrepancy in the declination reading after recentering is the equivalent of the declination drift for the time period (which can be calculated as the difference in the right ascension coordinates of the two reference stars used).

More formally:

$$\delta_{err} = \delta_2 - \delta_1$$
$$t = \alpha_0 - \alpha_1$$

Where:

δ_0, α_0 are the declination and right ascension coordinates of the first reference star.

δ_1, α_1 are the declination and right ascension coordinates of the second reference star.

δ_2, α_2 are the coordinates of the mount after recentering the second reference star.

Measuring Error by Using Plate Solving

In practice, measuring the declination deviation with stars is difficult to achieve with good accuracy. This is because any move in declination is subject to gear play, or backlash. Thus the resulting measurement may not account for declination deviation, but may also contain a backlash error component. Plate solving can help us solve that problem since no move in declination will be necessary.

Plate solving is an innovative technique whereby a digital image of the night sky is subjected to a plate-solving algorithm, which can determine, very accurately, the coordinates at the center of the image. Usually an estimate of these coordinates as well as the image scale are required as inputs to the algorithm.

Plate solving software can be used to automate measurement by moving only in right ascension. The procedure is similar to the manual procedure except that recentering the second reference star is not needed as the plate solve at the second location reveals the declination deviation. Here is the procedure:

1. Point the mount at a point near the optimal measurement location.
2. Take an image and plate solve its coordinates.
3. Synchronize the mount to these coordinates. As noted above it is important that the mount's pointing model be disabled for the procedure to work successfully.
4. Now move the mount in right ascension by some arbitrary amount. The movement should be short enough to remain close to the optimal measurement location, but be great enough to accurately measure the deviation. A movement of 15 to 30 minutes should suffice. The coordinates of the second location are simply: $\alpha_0 - t, \delta_0$, where t is the shift in right ascension and corresponds to the time of the measurement. Note that we do not want to move at all in declination.
5. Take a second image at this location and plate solve its coordinates. The declination difference between the second location and the plate solved coordinate is the deviation we seek: $\delta_{err} = \delta_p - \delta_0$, where δ_p is the declination coordinate from the plate solve.

Note that centering on a star is not required for either location. That is the beauty of plate solving. We can point at arbitrary points in the night sky and get very accurate coordinates.

Three Point Polar Alignment

In his 1879 work, *Lectures on Practical Astronomy and Astronomical Instruments*, Challis used declination drift measurements to determine the magnitude of both altitude and azimuth errors with a single star. [4] To do so requires three observations: 1) the initial position of the star, 2) the declination deviation and hour angle at some time later, and 3) the declination deviation and hour angle at a third point in time. Spherical trigonometry can then be used with these observations as inputs to determine the errors. The resulting math produces two equations in two unknowns (θ_{alt} and θ_{az}) which can be solved easily by any student of algebra. In 1879, Challis did not have the benefit of plate solving software. However, we should recognize that the same measurements that are needed to solve these equations can be obtained by plate solves along the same line of declination in three distinct locations.

Taki has extended Challis' work and gives a derivation of spherical trigonometric solutions to a 3-point measurement model using matrix methods in [5]. One particular appeal to this approach is that the math includes some modeling for the effects of atmospheric refraction which can add an edge to the accuracy of the method.

Moving Beyond the Drift Method

Measuring Error by Using Modeling

Mounts with pointing capability do an excellent job of modeling polar alignment errors. Although the methodology of creating a pointing model is beyond the scope of this article, it is mentioned here for completeness. A pointing model does not concern itself with declination drift per se, but is built up by pointing at two or more reference points (usually stars). As such it can usually model for errors beyond that of polar alignment. Such things as nonperpendicularity, flexure, mirror flop, and gear play can all be modeled and accounted for. This can lead to a more accurate estimate of the polar alignment error, as all of these things can affect the measurement of alignment error.

Two Star Polar Alignment

Alignment error can be determined by measuring the discrepancy when moving between the known coordinates of two carefully chosen reference points. The procedure works as follows. Point the telescope to a known location. This could be the coordinate of a star or the arbitrary coordinates determined by a plate solve. The mount is calibrated to this location (e.g. a sync operation). The telescope is moved to a second reference point. This could be a second star or an arbitrary move in right ascension and declination. The actual location where the optical axis is pointed is determined. This can be accomplished by recentering the star and noting the difference in coordinates or the coordinates can be resolved by means of a plate solve. The delta in right ascension and declination between the expected location and the actual location can be used to directly calculate the alignment error by means of spherical trigonometry. The interested reader is referred to Pass in [6] for the mathematics and deeper discussion.

Conclusion

Polar alignment error can best be observed at the celestial equator. The equatorial plane exhibits a tip or tilt due to the misalignments in altitude and/or azimuth. This tipping and tilting manifests itself as declination drift when a star is observed on or near the equator. This declination drift can be simulated more rapidly by moving the mount in right ascension, leading to some efficient techniques in measuring the alignment error. Finally, note that while declination drift techniques more directly reflect the theoretical geometry of misalignment, some may prefer modeling techniques because of the ability to account for other error terms. Being able to quantify the polar alignment error precisely allows us to know if greater precision is required. In more plain words, "When is good, good enough?" Quantifying the alignment error can also greatly increase the precision of the correction process.

Appendix A. Derivation of Alignment Error

Here we will show how declination drift can be used to measure the alignment error. First we start with Figure A1. This depicts the geometry of a misaligned axis at the celestial equator.

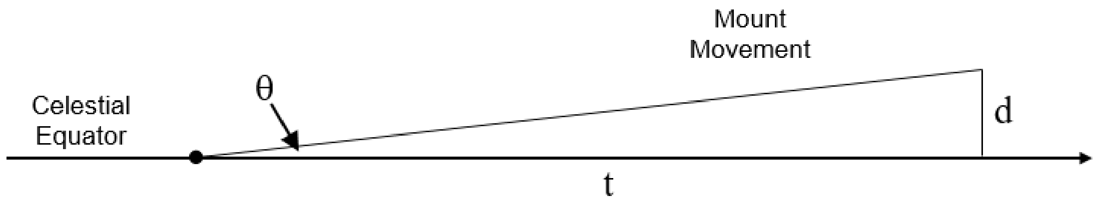

Figure A1. Geometry of a misaligned axis at the equator. θ is the angle formed by the celestial equator and the mount movement, d is the amount of declination drift which has been measured, and t is the time of the measurement (drift time, or movement in right ascension). All angles are measured in radians.

We can make some simplifying assumptions. First, we note that while the angles we are dealing with are actually on a sphere, since the angles are very small we can dispense with spherical trigonometry in lieu of planar geometry. In this case we can say:

$$\theta_r = \sin\left(\frac{d}{t}\right)$$

But again, since θ_r is small, we can also assume:

$$\theta = \sin(\theta), \quad \text{so, } \theta_r = \left(\frac{d}{t}\right)$$

The rest is a simple matter of unit conversions. Convert radians to degrees:

$$\theta_{\circ} = \left(\frac{180}{\pi}\right)\left(\frac{d}{t}\right)$$

To express time in sidereal minutes:

$$\theta_{\circ} = \left(\frac{180}{\pi}\right)\left(\frac{60 * d}{15 t_m \cos(\delta)}\right) = \left(\frac{720}{\pi}\right)\left(\frac{d}{t_m \cos(\delta)}\right)$$

To express the error angle in arc minutes:

$$\theta_{'} = \left(\frac{60 * 720}{\pi}\right)\left(\frac{d}{t_m \cos(\delta)}\right) = \left(\frac{43200}{\pi}\right)\left(\frac{d}{t_m \cos(\delta)}\right)$$

Finally, to express drift in arc seconds:

$$\theta_{'} = \left(\frac{43200}{3600 * \pi}\right)\left(\frac{d_{''}}{t_m \cos(\delta)}\right) = \left(\frac{12}{\pi}\right)\left(\frac{d_{''}}{t_m \cos(\delta)}\right)$$

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